Another method for achromatic of two lenses placed in contact

By lens maker's formula

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{---(i)}$$

$$\frac{1}{f(\mu-1)} = \left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{---(ii)}$$

By differentiating eqn (1) with respect to f

$$-\frac{1}{f^2} = \frac{d\mu}{df}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$-\frac{1}{f^2} = \frac{d\mu}{df} \times \frac{1}{f(\mu-1)}$$

$$-\frac{df}{f^2} = \frac{d\mu}{f(\mu-1)}$$

$$-\frac{df}{f} = \frac{d\mu}{\mu-1}$$

$$-\frac{df}{f'} = \frac{\omega}{f} \quad \text{---(iii)}$$

$$\frac{1}{F} = \frac{1}{f} + \frac{1}{f'}$$

$$\frac{d}{dF}\left(\frac{1}{F}\right) = \frac{d}{dF}\left(\frac{1}{f} + \frac{1}{f'}\right)$$

$$-\frac{1}{F^2} = -\frac{1}{f^2}\frac{df}{dF} - \frac{1}{f'^2}\frac{df'}{dF}$$

$$\frac{-dF}{F^2} = -\frac{df}{f^2} - \frac{df'}{f'^2}$$

$$-\frac{dF}{F^2} = \frac{\omega}{f} + \frac{\omega'}{f'} \quad \text{---(iv)}$$

Achromatism condition $dF = 0$

$$0 = \frac{\omega}{f} + \frac{\omega'}{f'}$$

$$\boxed{\frac{\omega}{f} = -\frac{\omega'}{f'}}$$

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